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MAY/JUNE 2014

## C ARIBBEAN <br> EXAMINATIONS <br> COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION ${ }^{\circledR}$
APPLIED MATHEMATICS
STATISTICAL ANALYSIS
UNIT 1 - Paper 02
2 hours 30 minutes

15 MAY 2014 (p.m.)

This examination paper consists of THREE sections: Collecting and Describing Data, Managing Uncertainty, Analyzing and Interpreting Data.

Each section consists of 2 questions.
The maximum mark for each section is 50 .
The maximum mark for this examination is 150 .
This examination consists of 9 printed pages and 2 answer sheets for Question 1 (f) (iv) and Question 2 (c).

## READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. DO NOT open this examination paper until instructed to do so.
2. Answer ALL questions from the THREE sections.
3. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.

## Examination Materials:

Mathematical formulae and tables (Revised 2010)
Electronic calculator
||| |||||||||||||||||||||||||
Ruler and graph paper

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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## SECTION A

## MODULE 1: COLLECTING AND DESCRIBING DATA

## Answer BOTH questions.

1. The management of The Family Boutique is concerned about the decrease in sales in the store. The following table shows the number of employees in the different departments, and the average weekly sales $(\times \$ 10000)$ for each department for the first quarter of the year.

|  | Department |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Men's <br> Clothing | Women's <br> Clothing | Children's <br> Clothing | Luggage <br> and Bag | Accessories | Jewellery and <br> Perfumes |
| Number of <br> Employees | 25 | 30 | 15 | 12 | 8 | 10 |
| Average <br> Weekly sales <br> $\mathbf{( \$ 1 0 ~ 0 0 0 ) ~}$ | 12 | 15 | 8 | 4 | 3 | 7 |

(a) (i) What does the 15 in the 'number of employees' row indicate?
[1 mark]
(ii) What does the 3 in the 'average weekly sales' row indicate?
[1 mark]
(b) Management has decided to select a random sample of 20 employees to get from them suggestions of what can be done to improve sales.

For EACH of the following sampling methods, state the name of the method that is being used.
(i) All the employees' names are put into a box, and 20 names are selected from the box.
[1 mark]
(ii) The employees' names are listed alphabetically. Starting with the third name, every fifth name thereafter is drawn, until 20 names are drawn.
(iii) Based on the proportional size of the department, a number of employees are randomly selected from EACH department to give a sample of 20 employees.
[1 mark]
(c) Which of the sampling methods listed in 1 (b) (i), (ii) and (iii) will result in a sample that is MOST representative of the employees from EACH department?
[1 mark]
(d) Using the method in 1 (b) (iii), calculate the number of employees in the sample that will be drawn from the Jewellery and Perfume department.
[3 marks]
(e) Calculate
(i) the average weekly sales for the entire store
(ii) the standard deviation of the sales for the entire store.
(f) The cumulative frequency graph below shows the time spent by 50 shoppers in the Men's Clothing Department of the Family Boutique on a particular day.

(i) Determine the number of shoppers who spent between 30 and 60 minutes in the Boutique.
[3 marks]
(ii) $60 \%$ of the shoppers spent $t$ minutes or less in the Boutique. What is the value of $t$ ?
(iii) Estimate the median time spent in the Boutique.
(iv) Using the graph sheet provided as an insert, draw a box-and-whisker diagram to represent the information given in the cumulative frequency graph.
[3 marks]

Total 25 marks
2. A construction company was setting up a new housing development in a certain country. Prospective home owners were invited to visit the site and discuss certain proposals with the contractors. The times, to the nearest minute, spent in consultation with 50 prospective home owners are summarized in the table below.

| Time $-x$ (minutes) | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of homeowners $-f$ | 3 | 14 | 22 | 10 | 1 |

(a) (i) State the boundaries of the third class of the distribution.
(ii) Calculate the size of the third class.
(iii) State ONE disadvantage of presenting data as a grouped frequency distribution.
[1 mark]
(b) Calculate the estimated

| (i) | mean time | [6 marks] |
| :--- | :--- | ---: |
| (ii) | variance | [4 marks] |
| (iii) standard deviation. | [2 marks] |  |

(c) On the graph paper provided, draw a histogram to represent the information given in the table.
[4 marks]
(d) Determine from the histogram, or otherwise, estimates for
(i) the mode of the distribution
[2 marks]
(ii) the number of consultations that lasted 45 minutes or more.
[2 marks]

Total 25 marks

## SECTION B

## MODULE 2: MANAGING UNCERTAINTY

## Answer BOTH questions.

3. (a) Of the total monthly production of an item, $60 \%$ are produced on machine A and the rest are produced on machine B. $2 \%$ of the items produced on machine A are defective, and $1 \%$ of the items produced on machine B are defective.
(i) Copy and complete the following tree diagram which shows this information, by inserting in the missing probabilities.

(ii) Calculate the probability that a randomly chosen item was produced by machine A and is defective.
(iii) Show that the probability that a randomly chosen item is defective is 0.016 .
(iv) Calculate the probability that a selected item came from machine A given that it is defective.
[3 marks]
(v) Two randomly selected items were tested. What is the probability that EXACTLY ONE of them is defective?
(b) Two events, $M$ and $N$, are such that $P(M)=0.6, P(M \cap N)=0.2, P(M \cup N)=0.85$

Calculate

| (i) | $P(N)$ | [2 marks] |
| :--- | :--- | ---: |
| (ii) | $P(N \mid M)$ | [2 marks] |
| (iii) | $P\left(M \cap N^{\prime}\right)$ | [2 marks] |

(c) State, with reason, whether $M$ and $N$ are
(i) mutually exclusive [2 marks]
(ii) independent. [2 marks]

## Total 25 marks

4. (a) State THREE conditions that will be necessary to model a binomial distribution.
[3 marks]
(b) Let $X$ be a binomial random variable with $n=12$ and $p=0.6$. Calculate
(i) $\quad P(X=3)$
[3 marks]
(ii) $\quad P(X \geq 2)$.
[4 marks]
(c) The lifespan of a certain insect is believed to follow a normal distribution with a mean of 72 days and a standard deviation of 8 days. Calculate the probability that an insect from this species will live for more than 84 days.
[5 marks]
(d) It is known that the probability that a particular variety of cabbage seed will germinate is 0.82 . A package contains 200 such seeds.
(i) Find the expected number of seeds from the package that will germinate.
(ii) Find a standard deviation of the numbers of seeds that will germinate.
[2 marks]
(iii) Use an approximate distribution to calculate the probability that less than 175 seeds from the package will germinate?

Total 25 marks

## SECTION C

## MODULE 3: ANALYZING AND INTERPRETING DATA

## Answer BOTH questions.

5. (a) The number of minutes that a sample of 12 students arrived late for a particular class were recorded as

| 3 | 13 | 5 | 8 | 12 | 5 | 6 | 4 | 8 | 7 | 10 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

(i) Calculate an unbiased estimator for the
a) mean number of minutes that students arrive late for class
b) variance of the number of minutes that students arrive late for class.
(ii) Assume that the number of minutes that students arrived late for the class follows a normal distribution.

A test was carried out at the $5 \%$ level of significance to determine whether the mean number of minutes that students arrived late was more than 7 minutes.
a) State, in statistical symbols, the null and alternative hypotheses.
b) Determine the critical region for the test.
c) Calculate the value of the test statistic.
d) Clearly state the conclusion of this test.
e) State the assumption that was made to perform this test. [1 mark]
(b) In a sample of 52 students, 18 arrived late for the class. Construct a $95 \%$ confidence interval for the proportion of students who arrived late for class.
[6 marks]

Total 25 marks
6. (a) Before students wrote a mathematics examination, their teacher had made predictions about the results that students will receive. After the actual results of the examinations, the results for 150 candidates were classified according to whether the predicted grade was better than, equal to, or worst than the actual grade, as shown in the following table.

|  |  | Predicted Grade |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Equal | Worse |  |  |
| Actual <br> Grade <br> Obtained | B | 16 | 30 | 12 | 58 |
|  | C | 10 | 20 | 9 | 39 |
|  | D | 20 | 15 | 18 | 53 |

A $\chi^{2}$ test at the $5 \%$ level of significance is carried out to determine whether the teacher's prediction and the actual results are independent.
(i) State the appropriate null and alternative hypotheses for this test. [2 marks]
(ii) Determine for the hypothesis test
a) the number of degrees of freedom [2 marks]
b) the critical region.
[2 marks]
(iii) Determine the expected frequency for the value 15 in the third row and the second column.
[2 marks]
(iv) The calculated $\chi^{2}$ value for this test is 9.1625.

Clearly state the conclusion that may be drawn from this test.
[2 marks]
(b) Salespersons at a certain establishment are required to take an aptitude test before starting the job. Six months later they are given a score based on their performance. For a sample of 10 such workers their scores on the aptitude test, $x$, and their corresponding performance scores, $y$, are summarized by the following:

$$
\Sigma x=377, \Sigma y=297, \Sigma x y=11305, \Sigma x^{2}=14397, \Sigma y^{2}=9145 .
$$

(i) Obtain the regression equation for $y$ on $x$ in the form $y=\mathbf{a}+\mathbf{b} x$. [6 marks]
(ii) Estimate the performance score for a person whose aptitude score was 37.
(iii) Interpret the value of $\mathbf{b}$ as it relates to the information given.
[2 marks]
(iv) Calculate the product-moment correlation, $r$, and interpret this value. [5 marks]

Total 25 marks

## END OF TEST

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Graph Sheet for Question 1 (f) (iv)
Candidate Number


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